Integrated Mathematics

An overview of the three-year cycle as designed for the adolescent aged 12-15 (grades 7-9)

The Montessori School of Winston-Salem—Adolescent Program
Integrated Mathematics Program Description

The Adolescent Program at The Montessori School of Winston-Salem builds upon the previous work of Elementary and Children’s House programs in fostering students’ strong development in mathematics: arithmetic, algebra, statistics, and geometry.

The adolescents’ mathematics experience provides both depth of study and breadth of exposure. It allows the adolescents to develop their ability to order their minds and think logically. Basic number concepts, arithmetic operations, and geometric ideas studied in earlier years are consolidated and applied through the students’ studies of topics in probability, basic statistics, applied geometry, and other areas. The students’ small business enterprise, meal preparation work, and “work on the land” provide a wealth of opportunities for the students to apply their math in real-world applications. Mathematics in the adolescent program is integrated throughout all the curriculum areas.

Adolescents have distinct developmental characteristics that are addressed in the mathematics program. First, their social needs dictate that they have the opportunity to work in a variety of group sizes and settings: individual, self-paced tutorials; small group collaborative projects; and whole-group seminars and lessons. Second, the adolescents need to do meaningful work of strong personal relevance. This is accomplished with arithmetic, geometry, statistics, and algebra applied in situations of interest to the adolescents: computation tips and budgets, creation of spreadsheets and income summaries, surveys of peers, design and layout of building projects, etc. Lastly, the adolescents need to know that they are informed of and meeting the basic norms of society. Therefore, they need regular and systematic lessons and practice, as well as training in note taking, study, and test-taking skills.

The mathematics curriculum is a logical continuation of the students’ work and studies in the Montessori Elementary program, follows the Montessori Adolescent Mathematics course work, and is closely aligned with that advocated by the American Association for the Advancement of Science, National Council of Teachers of Mathematics, and North Carolina State Standards. Upon graduation the TMS Adolescent, depending on the student’s individual needs, will be ready for either NC Math II or NC Math III course work.

There are four main components to the Adolescent Math Curriculum:

- Key concept lessons and skills practice
- Historical connections
- Application of skills through projects and work on the land
- Seminar discussions
- Residential life skills through budgeting, depreciation of assets, etc.
**Key Concepts and Skill Practice**
The adolescent will engage in small group and individual lessons where key concepts and ideas are presented. Each student will be allowed to learn and practice at his/her own pace while being supported by the adult guide. The use of direct instruction, continuous skills practice through Khan Academy, and regular ongoing teacher and self-assessment will provide the means for each adolescent to achieve mathematical competence.

**Historical Connections**
The adolescents will be able to make historical connections through the math history timelines. These timelines provide visual inspiration to learn about mathematicians and their important mathematical ideas and discoveries throughout the history. Students will use these timelines as a jumping off point to further investigate important mathematical equations and their creators. These learning opportunities will be through direct lessons, individual researches, as well as personal and group attempts to solve the great historical math problems.

**Application of Knowledge and Skills**
Adolescents need to feel that the work is meaningful and real otherwise they are not interested or engaged. Project work is the avenue to make math relevant for the adolescents. It will answer the age-old question of “When am I ever going to use this?” Through building projects, artistic endeavors, small business work, and group activities, the students will use their learned knowledge in ways that are directly related to their community work, be it in the classroom or on the land. These projects may include but are not limited to building a chicken coop, planning a walking path from the building to the barn, finding the volume of a fire pit, doubling a recipe for the community meal, or understanding a population graph that has grown exponentially over time.

**Math Seminar**
Adolescents need to discuss, debate, and share their thoughts and opinions. They need to work together while understanding that each person arrives at an answer in different ways. The math seminar is designed for the students to actively participate in finding the solution to a word problem. It requires them to think mathematically – to look for patterns and possible solutions for the problem and then practice the art of communicating and discussing their thought processes in a logical and clear way with their peers. The end goal is not what the correct answer is but rather the process the student went through in arriving at the answer – right or wrong. This activity requires the students to use the vocabulary and processes of math in ways that are meaningful and relevant.
Mathematical Tasks of the Adolescents

- Students learn essential vocabulary and communicate using mathematical rhetoric.
- Students develop an appreciation for all number systems.
- Students make connections between mathematical algorithms and the path needed to solve real world applications.
- Students learn how to write and solve equations, evaluate and simplify expressions.
- Students learn to use deductive and inductive reasoning, critical and lateral thinking to apply core concepts to applications and mathematical adventures.
- Students learn how to utilize previous knowledge in order to tackle advanced topics.

Ways that Math Tasks Are Achieved

- Frequent key lessons delivered by adult guides
- Frequent opportunities to practice and apply math in a variety of activities and projects
- Frequent opportunities to share work with peers and others through oral presentation, performance, or written publication
- Formal and informal group discussions about math concepts
- Formal seminars on math problems and ideas
- Support, practice, and remediation on skills if needed

Forms of Evaluation and Assessment

As is consistent with the Montessorian approach in the Upper and Lower Elementary programs at TMS, students actively participate in their own assessment and evaluation. Frequent self-evaluations, peer, and guide coaching sessions keep the adolescents apprised of their own progress in both efforts and achievements. Guides will also provide feedback and constructive criticism based on rubrics and other holistic forms of assessment to provide an adult perspective and foster their confidence in engaging in constructive dialogue about strengths and challenges in their work. There is a continued de-emphasis on letter grades; however, the students do receive feedback in the form of percentages. In the Ninth Year, end of course letter grades will be awarded to record progress for students’ high-school transcripts. Assessment tools include:

- Self-evaluations, including the use of rubrics and guiding questions
- Guide evaluations, using rubrics with specific standards
- Conferencing
- Ongoing coaching, discussion and feedback
- Quizzes and Tests
- Assigned questions and written reflection
- Math Seminar discussions
- Demonstration of mastery over subject matter as expressed through a variety of written and oral presentations, artistic expressions, or other culminating projects
- Portfolio development
- Application of mathematical skills
7th and 8th Year Math Concepts (Pre-Algebra – Math 1)

- Analyze proportional relationships and use them to solve real-world problems
- Apply and extend previous understandings of operations to solve problems with rational numbers
- Use properties of operations to generate equivalent expressions
- Solve real-life mathematical problems using numerical and algebraic expressions/equations
- Draw, Construct, and describe geometrical figures and describe relationships between them
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
- Use random sampling to draw inferences about a population
- Draw informal comparative inferences about two populations
- Investigate chance processes, develop/use/evaluate probability models
- Extend the properties of exponents to rational exponents
- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Create equations that describe numbers or relationships
- Understand solving equations as a process of reasoning and clearly explain reasoning using mathematics vocabulary
- Solve equations and inequalities in one variable and in two variables
- Solve systems of equations
- Represent and solve equations and inequalities graphically
- Understand the concept of a function and use of function notation
- Interpret functions that arise in applications in terms of context
- Analyze functions using different representations
- Build a function that models a relationship between two quantities
- Construct and compare linear and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model
- Use coordinates to prove simple geometric theorems algebraically
- Summarize, represent, and interpret discrete and continuous data with one variable
- Summarize, represent, and interpret data with two categorical and/or quantitative variables
- Interpret linear models algebraically and in context
9th Year Math Concepts (Math 2)

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers
- Define complex numbers
- Interpret the structure of expressions
- Perform arithmetic operations on polynomials
- Create multi-variable equations that describe numbers or relationships
- Understand solving equations as a process of reasoning
- Solve equations and inequalities in one variable and two variables
- Solve systems of equations
- Represent and solve equations and inequalities graphically
- Understand concept of a function and use function notation
- Interpret functions that arise in applications in terms of context
- Analyze functions using different representations
- Build a function that models a relationship between two quantities
- Build new functions from existing functions
- Experiment with transformations in the coordinate plane
- Understand congruence in terms of rigid motions in a coordinate plane
- Prove theorems in congruence
- Understand similarity in terms of similarity transformations in the coordinate plane
- Prove theorems using similarity
- Define Trigonometric ratios and solve problems involving right angles
- Understand and evaluate random processes underlying statistical experiments
- Understand independent and dependent (conditional) probability and use them to interpret data
- Conditional Probability and the Rules of Probability